

# PROBING THREE-BOSON ANOMALOUS COUPLINGS IN $e^+e^- \rightarrow W^+W^-$ AT FUTURE LINEAR $e^+e^-$ COLLIDERS WITH POLARIZED BEAMS

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## ABSTRACT

We study the potential of the next-generation  $e^+e^-$  linear colliders with longitudinally polarized beams, to restrict the values of the anomalous trilinear couplings  $WW\gamma$  and  $WWZ$  from the measurement of the process  $e^+e^- \rightarrow W^+W^-$ . Along with initial  $e^+e^-$  polarization, we account also for the possibilities offered by cross sections for polarized final  $W$ , in order to disentangle the constraints on the various constants. The results show the essential role of the initial beams polarization in improving the bounds obtained from the unpolarized case.

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## 1 Introduction

The experimental confirmation of the Standard Model (SM) is presently limited to the sector of the interaction of fermions with vector bosons. Another key ingredient of the SM is represented by the vector boson self-interactions, which are a consequence of the non-abelian structure of the electroweak symmetry, and are essential for the renormalizability of the theory.

The precise measurement of the three-boson  $WW\gamma$  and  $WWZ$  couplings is an important item in the physics programme at planned high energy (and high luminosity) colliders [1,2]. In the SM these vertices are exactly determined by the  $SU(2)_L \times U(1)_Y$  gauge symmetry, and therefore their measurement gives a unique chance to test the gauge structure of the electroweak theory. While experiments at low energy and precision measurements at the  $Z^0$  pole can provide *indirect* access to these constants, only very high energy colliders, well above the threshold for  $W$ -pair production, will allow *direct* and unambiguous tests. Indeed, in the near future one can foresee analyses of boson self-couplings at LEP2 [3,4] and to some extent at the Tevatron [4-7] and HERA [8].

A new stage in precision in this field will be reached at the planned hadron-hadron (SSC, LHC) and  $e^+e^-$  linear colliders (NLC, JLC, VLEPP), owing to the enhanced sensitivity to deviations from the SM, in particular to anomalous values of the gauge boson self-couplings, allowed by the significantly higher energies of these machines.

Among the various possible reactions, where to test the trilinear gauge boson couplings, a special role will be played by the process

$$e^+ + e^- \rightarrow W^+ + W^-, \quad (1)$$

at  $e^+e^-$  linear colliders [1,2,9-16]. This process should be particularly sensitive to deviations of the gauge boson couplings from the SM, originating from some “new physics” source. Such an enhancement of the sensitivity reflects the lack of compensation among the individual,  $s$ -diverging contributions to the SM cross section, corresponding at the Born level to  $\gamma$ ,  $\nu$  and  $Z$  exchange diagrams and their interferences. Instead, in the absence of new physics, such a gauge cancellation exactly occurs, and consequently the SM

cross section decreases with  $\sqrt{s}$  [17,18].

Indeed, in the specific case of modifications of the  $\gamma$ - and  $Z$ -mediated amplitudes, induced by anomalous values of the trilinear gauge boson couplings  $WW\gamma$  and  $WWZ$ , such that:

$$A_V \rightarrow (1 + \epsilon_V)A_V, \quad (2)$$

where  $V = \gamma, Z$ , we can define the relative deviation of the cross section of process (1) from the SM prediction as follows:

$$\Delta(s) = \frac{\sigma(s, \epsilon_V) - \sigma(s)^{SM}}{\sigma(s)^{SM}}. \quad (3)$$

Here,  $\sigma$  represents the cross section (either total, or differential, or integrated in some angular range), and

$$\begin{aligned} \sigma(s, \epsilon_V) &\propto |(1 + \epsilon_\gamma)A_\gamma + A_{1\nu} + (1 + \epsilon_Z)A_Z|^2 + |A_{2\nu}|^2, \\ \sigma^{SM}(s) &\propto |A_\gamma + A_{1\nu} + A_Z|^2 + |A_{2\nu}|^2, \end{aligned} \quad (4)$$

where for later convenience the neutrino-exchange amplitude is split into a part  $A_{1\nu}$  interfering with the  $s$ -channel diagrams and a non-interfering part  $A_{2\nu}$ , such that  $A_\nu = A_{1\nu} + A_{2\nu}$ . One obtains:

$$\Delta = \Delta_\gamma + \Delta_Z, \quad (5)$$

where

$$\begin{aligned} \Delta_\gamma &= \epsilon_\gamma(R_{\nu\gamma} + R_{Z\gamma} + 2R_{\gamma\gamma}), \\ \Delta_Z &= \epsilon_Z(R_{\gamma Z} + R_{\nu Z} + 2R_{ZZ}) \end{aligned} \quad (6)$$

and ( $i, j = \gamma, \nu, Z$ )

$$R_{ij} = \sigma_{ij}/\sigma^{SM}; \quad \sigma^{SM}(s) \equiv \sigma(s, \epsilon_V = 0) = \sum_{i,j} \sigma_{ij}. \quad (7)$$

Concerning the separation of the  $\nu$ -exchange diagram into  $A_{1\nu}$  and  $A_{2\nu}$ , introducing the helicities of  $W^-$  ( $W^+$ ) as  $\lambda$  ( $\bar{\lambda}$ ), the amplitude  $A_{2\nu}$  has  $|\Delta\lambda| = |\lambda - \bar{\lambda}| = 2$ , while all the others have  $|\Delta\lambda| = 0, 1$ . It turns out that  $A_{2\nu}$  is numerically dominant with respect to the other amplitudes, in the energy range considered below.

In Eqs.(5–7),  $\Delta$ 's are determined by linear combinations of non cancelling individually divergent contributions, and will increase, basically like a power of  $s$ . In contrast, the SM cross section decreases at least as  $1/s$ . Thus, if we parametrize the sensitivity of process (1) to  $\epsilon_V$  by *e.g.* the ratio  $\mathcal{S} = \Delta/(\delta\sigma/\sigma)$ , with  $\delta\sigma/\sigma$  the statistical uncertainty experimentally attainable on the SM cross section, such a sensitivity is power-like enhanced with increasing  $\sqrt{s}$ , even at fixed integrated luminosity.

An additional, and quite significant, improvement in the sensitivity should be obtained if initial  $e^+e^-$  longitudinal polarizations were available, so that one could separately measure the cross sections for both  $e_L^-e_R^+$  ( $\sigma^{LR}$ ) and  $e_R^-e_L^+$  ( $\sigma^{RL}$ ), as discussed in [19]. One should notice that  $\sigma^{RL}$  does not contain the neutrino-exchange diagram, which dominates the SM cross section and is not modified by the anomalous trilinear couplings. Therefore, this diagram represents a sort of “background” in the kind of searches discussed here. Consequently, although  $\sigma^{RL} \ll \sigma^{LR}$  leads a much lower statistics, in principle one can qualitatively expect to derive stringent limits in the  $RL$  case also. In fact, in practice longitudinal polarization will not exactly be 100%, and for realistic values of the polarization the determination of the  $RL$  cross section from the data could be totally obscured by the uncertainty in the polarization itself. Due to  $\sigma^{LR} \gg \sigma^{RL}$ , such an uncertainty could induce a systematic error on  $\sigma^{RL}$  much larger than the statistical error for this cross section, and consequently the sensitivity would be diminished. However, as it will be discussed in the sequel, one can find “optimal” kinematical regions to integrate cross sections, where this effect does not so dramatically contribute to the uncertainty on the  $RL$  cross section, and therefore the expected sensitivity on the anomalous coupling constants provided by  $\sigma^{RL}$  qualitatively remains the same. Clearly, as in [19], the complete analysis should combine measurements of  $\sigma^{RL}$  and  $\sigma^{LR}$ , in particular for the purpose of disentangling the constraints for the different anomalous vertices.

In this regard, combined measurements of cross sections for final polarized (longitudinal and/or transverse)  $W$ ’s with polarized initial beams would also be extremely useful both to improve the sensitivity to individual anomalous couplings and to separate the various dependences.

In this paper we will present the bounds on the anomalous three-boson constants, which can be obtained along the lines sketched above from the consideration of the process  $e^+e^- \rightarrow W^+W^-$  at future high-luminosity linear  $e^+e^-$  colliders, with CM energies of  $0.5 - 1 \text{ TeV}$  and polarized beams, assuming that also  $W^+W^-$  polarizations will be measured.

Specifically, in Section 2 we will introduce the standard parameterization of the  $WW\gamma$  and  $WWZ$  vertices in terms of the familiar notation using  $k_V$  and  $\lambda_V$ , and will briefly review the current bounds on these parameters as well as the expectations from forthcoming experiments. In Section 3 we analyze in details the potential of process (1) and the role of polarization, and assess the resulting constraints on the anomalous couplings. Finally, Section 4 will be devoted to a discussion of the results and to some concluding remarks.

## 2 WWV vertices

Limiting to the  $C$  and  $P$  invariant part of the interaction,<sup>2</sup> the  $WWV$  coupling, represented in Fig.1, is [9]:

$$\mathcal{L}_{eff} = -ig_{WWV} \left[ W_{\mu\nu}^+ W^\mu V^\nu - W_\mu^+ V_\nu W^{\mu\nu} + k_V W_\mu^+ W_\nu F^{\mu\nu} + \frac{\lambda_V}{M_W^2} W_{\lambda\mu}^+ W_\nu^\mu F^{\lambda\nu} \right]. \quad (8)$$

Here,  $W^\mu$  is the  $W^-$  boson field,  $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ ,  $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ , and the gauge coupling constants  $g_{WWV}$  are, with  $V = \gamma, Z$

$$g_{WW\gamma} = e; \quad g_{WWZ} = e \cdot \cot\theta_W, \quad (9)$$

where  $e$  is the electron charge and  $\theta_W$  the electroweak mixing angle.

For the  $WW\gamma$  coupling in the static limit there is a simple interpretation of the parameters appearing in Eq.(8), namely  $g_{WW\gamma}$  defines the  $W$  electric charge, while  $k_\gamma$  and  $\lambda_\gamma$  are connected with the magnetic ( $\mu_W$ ) and electric quadrupole ( $Q_W$ ) moments of the  $W$  boson:

$$\mu_W = \frac{e}{2M_W}(1 + k_\gamma + \lambda_\gamma) \quad , \quad Q_W = -\frac{e}{M_W^2}(k_\gamma - \lambda_\gamma). \quad (10)$$

A similar interpretation holds for the parameters  $k_Z$  and  $\lambda_Z$  in the  $WWZ$  vertex.

Referring to Fig.1, in momentum space the vertex of Eq.(8) can be written as [9]:

$$\Gamma_{\mu\alpha\beta}^V(q, \bar{q}, p) = (\bar{q} - q)_\mu \left[ \left(1 + \frac{\lambda_V p^2}{2M_W^2}\right) g_{\alpha\beta} - \lambda_V \frac{p_\alpha p_\beta}{M_W^2} \right] + (p_\beta g_{\mu\alpha} - p_\alpha g_{\mu\beta})(1 + k_V + \lambda_V). \quad (11)$$

In the SM at the tree level,  $k_V = 1$  and  $\lambda_V = 0$ . The existing limits on  $k_V$  and  $\lambda_V$  are rather loose,  $\leq O(1)$ . Bounds on the anomalous moments can be derived from high-precision measurements of electroweak observables which are affected by  $W$ -loop corrections [21,22]. These effects have been discussed for the  $Z$  boson parameters and for atomic parity violation. From low-energy data and from precision LEP I measurements:

$$\begin{aligned} |\lambda_\gamma| &\leq 0.6 & |k_\gamma - 1| &\leq 1.0 \\ |\lambda_Z| &\leq 0.6 & -0.8 \leq k_Z - 1 &\leq 0 \end{aligned}$$

However, some combinations of  $k_V$  and  $\lambda_V$  can be restricted more tightly: for example,  $|\lambda_\gamma - \lambda_Z| \leq 0.1$  for any value of  $\lambda_{\gamma,Z}$  and  $|\lambda_\gamma - \lambda_Z| \leq 0.01$  for  $\lambda_{\gamma,Z} > 0.25$ . In perspective, from the  $q\bar{q} \rightarrow W\gamma$  and  $q\bar{q} \rightarrow WZ$  processes at the Tevatron with an integrated luminosity of  $1 \text{ fb}^{-1}$ , one can obtain the following bounds:

$$\begin{aligned} |\lambda_\gamma| &\leq 0.2 & -0.50 \leq k_\gamma - 1 &\leq 0.80 \\ |\lambda_Z| &\leq 0.4 & -0.80 \leq k_Z - 1 &\leq 0 \end{aligned}$$

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<sup>2</sup>In this paper we do not consider  $C$ ,  $P$  and  $T$  violating operators. The latter should be strongly suppressed, *e.g.* by the upper bound on the neutron electric dipole moment, which implies for the  $WW\gamma$  vertex  $|\tilde{k}_\gamma|, |\tilde{\lambda}_\gamma| \leq O(10^{-4})$  [20]. We also neglect a possible deviation of the Yang-Mills coupling constants  $g_{WWV}$  from those predicted by the SM.

The corresponding limits at LEP II with  $\sqrt{s} \simeq 200$  GeV will be [3,4]:

$$\begin{aligned} |\lambda_\gamma| &\leq 0.4 & -0.14 \leq k_\gamma - 1 \leq 0.87 \\ |\lambda_Z| &\leq 0.4 & -0.24 \leq k_Z - 1 \leq 0 \end{aligned}$$

For the higher energy  $e^+e^-$  linear colliders, the study of the process  $e^+e^- \rightarrow W^+W^-$  with no initial longitudinal polarization can lead to limits on  $|\lambda_V|$  and  $|k_V|$  typically of the order of some *percent* [1,2]. Regarding the separation of  $k_V$  from  $\lambda_V$ , the advantages of using the processes  $e^-\gamma \rightarrow W^-\nu$ ,  $\gamma\gamma \rightarrow W^+W^-$  and  $e^+e^- \rightarrow \nu\bar{\nu}Z(\rightarrow \mu^+\mu^-)$  have been investigated in the literature [11,12,23-26]. However, for these processes the sensitivity to anomalous trilinear couplings turns out not to be improved with respect to the numbers reported above. Also, restrictions similar to those found at linear  $e^+e^-$  colliders could be obtained from the complementary studies at the hadron-hadron supercolliders [4-7].

In the next Section we are going to discuss in details the potential of process (1) to constrain  $\lambda_V$  and  $k_V$ , particularly emphasizing the role of *initial*, in addition to final states polarizations.

### 3 The process $e^+e^- \rightarrow W^+W^-$ with polarization

The  $e^+e^-$  annihilation into a  $W$ -pair is determined in Born approximation by the diagrams in Fig.2. We start our discussion with the case of initial  $RL$  polarization, and with final longitudinal  $W_LW_L$  or transverse  $W_TW_T$  polarizations. In the sequel these two cases for the  $W$  polarizations will be denoted by  $LL$  and  $TT$ , respectively. The corresponding transition amplitudes can be written as (see Appendix):

$$\mathcal{A}_{LL}^{RL} = \frac{s}{M_W^2} \left[ \frac{3 - \beta_W^2}{2} (1 - \chi \cdot g_Z g_R) + (\Delta k_\gamma - \chi \cdot g_Z g_R \Delta k_Z) \right], \quad (12)$$

and

$$\mathcal{A}_{TT}^{RL} = (1 - \chi \cdot g_Z g_R) + \frac{s}{2M_W^2} (\lambda_\gamma - \chi \cdot g_Z g_R \lambda_Z). \quad (13)$$

Here and in the following the notations are such that upper indices refer to initial  $e^-e^+$  longitudinal polarizations, and the lower indices indicate the final  $W^\pm$  longitudinal and/or transverse polarizations. In (12) and (13), which are easily derived using Table 3.1 of Ref.[1]:  $\Delta k_V = k_V - 1$  and  $\lambda_V$  are the anomalous trilinear couplings as defined in (8);  $\beta_W = \sqrt{1 - 4M_W^2/s}$ ;  $g_Z = \cot \theta_W$ ;  $g_R = \tan \theta_W$  is the right-handed electron coupling constant; and finally  $\chi = s/(s - M_Z^2)$  is the  $Z$  propagator. The explicit expressions of  $\epsilon_V$  introduced in Eq.(2) can be easily obtained for the various polarizations from Eqs.(12)–(13). Notice that in these equations we have not explicitly included an angular-dependent factor  $\propto \sin \theta$ , common to all s-channel helicity amplitudes.

In terms of the amplitudes (12) and (13), the total cross sections, integrated over all angles, are given by:

$$\sigma_{LL}^{RL} = \frac{\pi\alpha_{e.m.}^2\beta_W^3}{6s}|\mathcal{A}_{LL}^{RL}|^2, \quad (14)$$

and

$$\sigma_{TT}^{RL} = \frac{4\pi\alpha_{e.m.}^2\beta_W^3}{3s}|\mathcal{A}_{TT}^{RL}|^2. \quad (15)$$

One can notice that  $\sigma_{LL}^{RL}$  and  $\sigma_{TT}^{RL}$  separately depend on different sets of trilinear gauge boson couplings, and therefore give independent information on  $k$ 's and  $\lambda$ 's. In contrast, the  $LT+TL$  cross section turns out to depend on all four couplings and therefore is not so useful for disentangling their effects. Detailed formulae and definitions are collected in the Appendix, which can also be used to derive explicit expressions for the deviations from the SM due to anomalous couplings, defined in Eqs.(6)–(7), for the different polarizations.

In Fig.3 we represent the energy behavior of  $\sigma_{LL}^{RL}$  and  $\sigma_{TT}^{RL}$  in the SM, along with the cross section for unpolarized  $W$  bosons. The latter case includes the sum over the three possibilities  $LL$ ,  $TT$  and  $TL+LT$ . In accordance to Eqs.(14)–(15), Fig.3 shows that the  $LL$  cross section is largely dominating for increasing energy over the  $TT$  cross section, by a factor of order  $(s/M_W^2)^2$ . Consequently, sensitivities on anomalous couplings expected from statistical arguments, denoted as  $\mathcal{S}$  in Section 1, will have the behavior  $\mathcal{S}_{LL} \propto s^{3/2}$  and, provided  $\sigma_{TT}^{RL}$  could be measured,  $\mathcal{S}_{TT} \propto s^{5/2}$ . Anyway, as discussed later in this Section, although much smaller than in the  $LL$  case, the  $TT$  cross section could be useful in order to constrain the values of the couplings  $\lambda_\gamma$  and  $\lambda_Z$ . Also, observing from Fig.3 that the  $LL$  cross section is numerically close to the unpolarized one, we obviously conclude that the latter cross section is mostly sensitive to  $\Delta k_V$ , rather than to  $\lambda_V$ .

To derive the typical values of the bounds on anomalous couplings, that can be derived from  $e_L^+e_R^- \rightarrow W^+W^-$ , we choose two possible values of the CM energy, referring to the planned NLC colliders [27], namely  $\sqrt{s} = 0.5 \text{ TeV}$  and  $\sqrt{s} = 1 \text{ TeV}$ , with integrated luminosities  $L_{int} = 50 \text{ fb}^{-1}$  and  $L_{int} = 100 \text{ fb}^{-1}$ , respectively. In both cases we use the channel of two leptons plus two hadronic jets ( $l \nu + j j$ ) to identify the final  $W^+W^-$  state, with an efficiency of reconstruction  $\varepsilon_W = 0.15$ . Referring to Eq.(3), in this Section we discuss the bounds which would be derived from just statistical arguments, *i.e.* by demanding that

$$|\Delta| = \frac{|\sigma(\Delta k_V, \lambda_V) - \sigma^{SM}|}{\sigma^{SM}} < \frac{\delta\sigma}{\sigma}, \quad (16)$$

with  $\delta\sigma/\sigma$  the statistical uncertainty in the specific cases. We defer to the next Section a presentation of the bounds taking into account also some systematic uncertainties.

Then, by solving the inequality (16) in the case of  $\sigma_{LL}^{RL}$ , we obtain the two conditions:

$$|\Delta k_\gamma - \chi \cdot \Delta k_Z| < \frac{1}{2} \left( \frac{\delta\sigma}{\sigma} |\tilde{\mathcal{A}}| \right)_{LL}^{RL}, \quad (17)$$

and

$$|\Delta k_\gamma - \chi \Delta k_Z + 2\tilde{\mathcal{A}}_{LL}^{RL}| < \frac{1}{2} \left( \frac{\delta\sigma}{\sigma} |\tilde{\mathcal{A}}| \right)_{LL}^{RL}, \quad (18)$$

where  $\tilde{\mathcal{A}}_{LL}^{RL} = \frac{3 - \beta_W^2}{2}(1 - \chi)$ . Eqs.(17) and (18) are derived under the assumption that the statistical uncertainty  $\delta\sigma/\sigma \ll 1$ , so that one can expand (16) to first order in this parameter. As seen from Fig.3, in the chosen range of  $\sqrt{s}$  the values of the SM cross sections are  $\sigma_{LL}^{RL} = 84 \text{ fb}$  and  $22 \text{ fb}$  for  $\sqrt{s} = 0.5 \text{ TeV}$  and  $1 \text{ TeV}$ , respectively. Correspondingly, from (14), for the right-hand sides of Eqs.(17)–(18) we find the values:

$$\frac{1}{2} \left( \frac{\delta\sigma}{\sigma} |\tilde{\mathcal{A}}| \right)_{LL}^{RL} = \frac{1}{\sqrt{(\sigma_0)_{LL} \varepsilon_W L_{int}}} \cdot \frac{M_W^2}{s} = 1.3 \cdot 10^{-3} \text{ (} 4.6 \cdot 10^{-4} \text{)}, \quad (19)$$

at the 95% C.L. for the two values of the CM energy, with  $(\sigma_0)_{LL} = \pi \alpha_{e.m.}^2 \beta_W^3 / 6s$ .

In Fig.4 the bands labeled as ‘1’ and ‘2’ represent the allowed domains for the anomalous constants, resulting from the inequalities above.<sup>3</sup> At this stage, since the bands are not limited, we have no restrictions on  $\Delta k_\gamma$  and  $\Delta k_Z$  separately, but only correlations between their values. To obtain separate bounds one has to change the *slopes* of the bands, in such a way as to find new bands crossing the preceding ones ‘1’ and ‘2’. To this purpose, referring to the left sides of Eqs.(17) and (18), there are two possibilities, namely either to exploit the *s*-behavior of the *Z* propagator  $\chi$ , affecting the slopes, or to change the initial beams polarization, and consider *e.g.* the *LR* cross section. Since at the considered energies  $\chi$  is weakly dependent on *s*, only the latter possibility remains.

Turning therefore to the case of *LR* initial longitudinal polarizations, the analogue of Eq.(12) reads:

$$\mathcal{A}_{LL}^{LR} = \frac{s}{M_W^2} \left[ \tilde{\mathcal{A}}_{LL}^{LR} + (\Delta k_\gamma - \chi \cdot g_Z g_L \Delta k_Z) \right], \quad (20)$$

where

$$\tilde{\mathcal{A}}_{LL}^{LR} = \tilde{\mathcal{A}}_{LL}^{LR}(\nu) + \tilde{\mathcal{A}}_{LL}^{LR}(\gamma, Z), \quad (21)$$

$$\tilde{\mathcal{A}}_{LL}^{LR}(\gamma, Z) = \frac{3 - \beta_W^2}{2} (1 - \chi \cdot g_Z g_L). \quad (22)$$

Referring to the separation of the neutrino exchange amplitude introduced in Eq.(4), one may notice that only the amplitude  $A_{1\nu}$  appears in Eq.(21). The *LR* integrated cross section can be obtained similar to Eq.(14), using the formulae given in the Appendix. Also in this case one finds two allowed bands for  $\Delta k_\gamma$  and  $\Delta k_Z$ . However, taking into account that  $g_Z g_L = -1.17$  (or  $-1$  for  $\sin^2 \theta_W = 0.25$ ), as opposed to  $g_Z g_R = +1$ , these bands limit the combination  $|\Delta k_\gamma + 1.17\chi \cdot \Delta k_Z|$ , and are thus almost orthogonal to the bands previously derived in the *RL* case for the combination  $|\Delta k_\gamma - \chi \cdot \Delta k_Z|$ , see Eqs.(17)

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<sup>3</sup>If we account for the imaginary part of the propagator, *i.e.*  $\chi \rightarrow s/(s - M_Z^2 + iM_Z\Gamma_Z)$ , the straight bands transform into a region enclosed by two ellipses.



and (18). Concerning the widths of the  $LR$  and the  $RL$  bands, by actual numerical calculations these are found to be qualitatively of the same size, provided one limits the angular integration of  $\sigma^{LR}$  to a range including the backward hemisphere, for example the range  $-0.9 \leq \cos \theta \leq 0.3$  which will be considered later on for a more quantitative analysis. The lower limit of integration is chosen from experimental conditions, while the upper one minimizes the “background” from the neutrino exchange diagram, which dominates in the forward direction and does not contain the trilinear coupling constants, thus reducing the sensitivity in the forward hemisphere [19]. Corresponding to this integration range, one obtains the bands labelled in Fig.4 as ‘3’ and ‘4’, which together with ‘1’ and ‘2’ determine four allowed regions for the coupling constants, respectively labeled as  $a$ ,  $b$ ,  $c$ ,  $d$ . Since numerically  $\tilde{\mathcal{A}}_{LL}^{LR}(\nu) < 0$ ,  $\tilde{\mathcal{A}}_{LL}^{LR}(\gamma, Z) > 0$ , with  $|\tilde{\mathcal{A}}_{LL}^{LR}(\nu)| > |\tilde{\mathcal{A}}_{LL}^{LR}(\gamma, Z)|$ , one can see that  $\tilde{\mathcal{A}}_{LL}^{LR} < 0$  (see Eq.(21)) shifts the position of the band ‘4’ upwards with respect to the band ‘3’. Note that only the region  $a$  in Fig.4 is compatible with the SM values  $\Delta k_Z = \Delta k_\gamma = 0$ .

While the “ideal” situations presented above assumed 100%  $RL$  or  $LR$  polarizations, in practice the cross section is expressed as:

$$\sigma = \frac{1}{4} \left[ (1 + P_1) \cdot (1 - P_2) \sigma^{RL} + (1 - P_1) \cdot (1 + P_2) \sigma^{LR} \right], \quad (23)$$

where  $P_1$  ( $P_2$ ) are less than unity, and represent the actual degrees of longitudinal polarization of  $e^-$  ( $e^+$ ). For any two different couples of values for  $(P_1, P_2)$ , there will be four intersections, similar to Fig.4. Thus, by varying  $P_1$  and  $P_2$ , the slopes of the bands in Fig.4 change, in such a way that the allowed region corresponding to  $a$  remains in the same position (around the origin), and the other intersections change their positions in the  $(\Delta k_\gamma, \Delta k_Z)$  plane, and therefore should be excluded. Accordingly, one would need to perform experiments at (at least) two different pairs of suitable values of  $P_1$  and  $P_2$ , in order to reduce the allowed regions to just region  $a$ . Actually, one can easily see that the extension of this region  $a$  would be minimized by the symmetric choice  $P_1 = -P_2 = P > 0$  and  $P_2 = -P_1 = P > 0$ , if experimentally feasible.

Combining Fig.4 with Eq.(19) and the subsequent discussion on  $\sigma_{LL}^{RL}$  and  $\sigma_{LL}^{LR}$ , we qualitatively obtain that the expected constraints on the anomalous coupling constants could be of the order of  $|\Delta k_V| < 10^{-3}$  ( $10^{-4}$ ) for the two considered values of  $\sqrt{s}$ .

As it was mentioned in Section 1, in principle the uncertainty on the polarizations  $P_1$  and  $P_2$  can induce a large systematic uncertainty on the determination of  $\sigma^{RL}$ , because  $\sigma^{LR} \gg \sigma^{RL}$  for total cross sections. The effect of this contamination can be substantially reduced by observing that the bulk of  $\sigma^{LR}$  comes from the forward direction, and that the ratio between  $\sigma^{RL}$  and  $\sigma^{LR}$  is not so negligibly small in the backward direction, say  $\cos \theta < 0$ , as exemplified in Fig.5. This suggests that one should limit to just this kinematical region, without substantial loss on the statistics for  $\sigma^{RL}$ . In this kinematical region

$\sigma^{LR}/\sigma^{RL} = 10-20$ , so that, with *e.g.*  $P_1 = -P_2 = 0.8$ , and with  $\delta P_1/P_1 = \delta P_2/P_2 = 0.01$ , such an uncertainty on the polarization induces on  $\sigma^{RL}$  a *systematic* uncertainty of the order of  $(\delta\sigma/\sigma)_{sys}^{RL} \simeq 10^{-2}$ . This is to be compared to a statistical uncertainty on the  $RL$  cross section (in the backward region) of about  $6 \cdot 10^{-2}$  (at  $\sqrt{s} = 0.5 \text{ TeV}$ ).

We turn now to the process of transversely polarized  $W^\pm$  production for different polarizations of the initial  $e^+e^-$  beams, and first compare its features with the case, considered previously, of  $W_L^+ W_L^-$  production. In particular, as noticed above with regard to Fig.3, the corresponding SM cross section  $\sigma_{TT}^{RL}$  at the energies of interest here is too small to provide sufficient statistics. Indeed,  $\sigma_{TT}^{RL} = 0.4 \text{ fb}$  ( $7 \cdot 10^{-3} \text{ fb}$ ) at  $\sqrt{s} = 0.5 \text{ TeV}$  ( $1 \text{ TeV}$ ). Consequently, for a one year run we expect only three (less than one) event samples. On the other hand, as can be seen from Eq.(13), the contributions coming from anomalous terms increase the cross section due to the enhancement factor  $s/2M_W^2$  contained in amplitude. Therefore, although the SM cross section could not be observable at a given level of accuracy, the cross section with large enough values of the anomalous vertices might be observable at high energy. Thus, to the purpose of deriving restrictions on the anomalous couplings from transversally polarized  $W^\pm$  production, we can adopt as a criterion for observability of  $TT$  events, the condition that the relevant cross section should be observable with an uncertainty

$$\delta\sigma_{TT}^{RL}(\lambda_V) < \sigma_{TT}^{RL}(\lambda_V) - \sigma_{TT}^{RL}(SM). \quad (24)$$

The reverse of this inequality will give upper limits on  $\lambda_V$ . We can notice that in this case the criterion is different from the one in Eq.(16), where the SM cross section is assumed to be measured. Actually, it would conceptually coincide with Eq.(16) if  $\sigma(SM)$  was measurable with good statistics, whereas for  $\sigma(SM) \ll \sigma(\lambda_V)$  it requires that at least four  $W_T^- W_T^+$  production events are observed in order to be able to state that  $\sigma(\lambda_V) \neq 0$  at the 95% C.L.

The comparison between the amplitudes of process (1) for the production of longitudinally and transversely polarized  $W^\pm$  pairs shows that, due to their similar dependence on anomalous couplings (see Eqs.(12) and (13)), the expected bounds on  $\lambda_V$  in the  $(\lambda_\gamma, \lambda_Z)$  plane should qualitatively have the same form as the band ‘1’ in Fig.4, previously derived in the  $(W_L^+ W_L^-)$  case from Eq.(17). Indeed, from Eq.(13) and the above criterion, for the  $RL$  case one has (assuming  $\sigma^{SM} \ll \sigma(\lambda_V)$ ) the upper limit

$$|\lambda_\gamma - \chi \cdot \lambda_Z| < \frac{4}{\sqrt{(\sigma_0)_{TT} \varepsilon_W L_{int}}} \cdot \frac{M_W^2}{s}, \quad (25)$$

which expresses the 95% C.L. bounds. In (25), we denote  $(\sigma_0)_{TT} = 4\pi\alpha_{e.m.}^2 \beta_W^3 / 3s = 8(\sigma_0)_{LL}$ . Contrary to the case of Eqs.(17) and (18), we have here just one inequality, because we consider only positive sign for the deviation on right side of Eq.(24). For

the inputs used in the previous cases, the typical values of the right side of Eq.(25), characterizing the width of the band in the  $(\lambda_\gamma, \lambda_Z)$  plane labeled as ‘1’ in Fig.6, are  $1.8 \cdot 10^{-3}$  ( $6.5 \cdot 10^{-4}$ ) at  $\sqrt{s} = 0.5 \text{ TeV}$  ( $1 \text{ TeV}$ ).

At this point, analogous to the case studied above of longitudinally polarized  $W^\pm$  bosons, the  $LR$  polarization of the initial  $e^-e^+$  beams with final transversely polarized  $W$ ’s could be used to change the slope of the band obtained from Eq.(25) and turn it into a band limiting  $|\lambda_\gamma + 1.17\chi \cdot \lambda_Z|$ . A finite domain allowed to  $\lambda_\gamma$  and  $\lambda_Z$  should then occur from the combination of the two bands. However, in contrast to the  $W_L^+W_L^-$  case, where the widths of  $RL$  and  $LR$  numerically turn out to be comparable, the  $\nu$ -mediated amplitude for production  $W_T^+W_T^-$  has the additional part  $A_{2\nu}$  with  $\Delta\lambda = \pm 2$ . This amplitude does not interfere with the  $s$ -channel amplitudes (see Eq.(4)), and at the same time significantly increases the SM cross section as well as the cross section with anomalous couplings. Recalling Eq.(4) and the criterion (24), one can see that the presence of  $A_{2\nu}$  affects the limits on  $|\lambda_\gamma + 1.17\chi \cdot \lambda_Z|$  by a factor  $\sqrt{1 + (\sigma_{2\nu}/\sigma_1)}$ , where  $\sigma_{2\nu} \propto |A_{2\nu}|^2$  and  $\sigma_{1\nu} \propto |A_\gamma + A_{1\nu} + A_Z|^2$ . Numerically,  $\sigma_{2\nu}$  by far dominates over the  $\Delta\lambda = 0$  cross section  $\sigma_{1\nu}$ , and dramatically increases the factor mentioned above and the corresponding width of the allowed band for  $|\lambda_\gamma + 1.17\chi \cdot \lambda_Z|$ . Only in the specific kinematical region of outgoing  $W^-$  in the backward direction, the ratio of cross sections (integrated over the backward hemisphere) can reduce to as low as  $\sim 10^2$ , but not less. In this case the width of  $LR$  band would be approximately ten times larger than the  $RL$  one, as it is shown in Fig.6. One can see that the width of band ‘1’ allowed by  $\sigma_{TT}^{RL}$  is of the order of  $10^{-3}$ , whereas band ‘2’ allowed by  $\sigma_{TT}^{LR}$  has a width of the order of  $10^{-2}$ . Consequently, we can conclude that the anomalous couplings  $(\lambda_\gamma, \lambda_Z)$  can be strongly correlated, rather than being severely restricted in a symmetric small region. For  $\sqrt{s} = 1 \text{ TeV}$ , and the corresponding assumed luminosity, the limits of Fig.6 are improved by a factor  $\sqrt{L_{int}s} = \sqrt{8}$ .

## 4 Discussion and concluding remarks

The integrated and differential cross-sections as well as the asymmetries of the process under consideration are commonly considered as the basic experimental observables to study deviations from the SM induced by new physical effects. Continuing our previous discussion of the sensitivity of the  $e^+e^- \rightarrow W^+W^-$  process to the anomalous  $WW\gamma$  and  $WWZ$  couplings, we will use, for the different cases corresponding to specific initial and final polarizations, the integrated cross section  $\sigma(z_1, z_2)$  and the forward-backward asymmetry  $A_{FB}$ , respectively defined as ( $z \equiv \cos\theta$ ):

$$\sigma(z_1, z_2) = \int_{z_1}^{z_2} \frac{d\sigma}{dz} dz; \quad A_{FB} = \frac{\sigma(0, z_2) - \sigma(z_1, 0)}{\sigma(z_1, z_2)}. \quad (26)$$

To work out an example closer to the realistic situation, which somehow could account for both statistical and systematic experimental uncertainties, we assume a systematic error in the cross-section measurement at the level of  $\sim 2\%$  [28], resulting from an uncertainty in the luminosity measurement ( $\delta L_{int} \simeq 1\%$ ), an error in the acceptance ( $\delta_{accept} \simeq 1\%$ ), an error for background subtraction ( $\delta_{backgr} \simeq 0.5\%$ ), a systematic error on the knowledge of the branching ratio of  $W \rightarrow \bar{f}f$  ( $\delta_{Br} \simeq 0.5\%$ ). Finally, we assume the degrees of longitudinal polarizations  $|P_1|, |P_2| = 0.8$ , with an uncertainty  $\delta P_1/P_1 = \delta P_2/P_2 = 10^{-2}$ . We notice that for the integrated luminosities assumed in the previous Section,  $L_{int} = 50 \text{ fb}^{-1}$  and  $100 \text{ fb}^{-1}$  at  $\sqrt{s} = 0.5 \text{ TeV}$  and  $1 \text{ TeV}$  respectively, the systematic uncertainty would dominate over the statistical one in the case of initial  $LR$  and unpolarized  $e^-e^+$ , whereas the converse would be true for initial  $RL$  due to the smallness of this cross section. The numerical results for the bounds on the anomalous couplings presented in this Section are derived using a combined  $\chi^2$  analysis of the integrated cross sections and of the forward-backward asymmetry. All allowed domains will be given to 95% C.L., corresponding to  $\Delta\chi^2 = 5.99$  for two simultaneously fit free parameters.

In general, we have four free couplings to fit to the experimental data. For unpolarized initial and final states, it is therefore not possible to disentangle the dependence of the cross section on the various constants. The simplest procedure is to fix a couple of parameters at the SM values, and derive allowed regions for the remaining ones. In Figs.7a,b we fix  $\lambda_Z, k_Z$ , and consider cross sections at  $\sqrt{s} = 0.5 \text{ TeV}$ , integrated in the “optimal” range  $-0.9 \leq z \leq 0.3$  already introduced in the previous Section, where anomalous effects are most pronounced. The regions allowed to  $\lambda_\gamma, k_\gamma$  by such a procedure are the ones enclosed by the dashed ellipses in Fig.7a for unpolarized beams, while initial longitudinal  $e^-e^+$  “ $LR$ ” polarization with  $P_1 = -P_2 = -0.8$  provides the allowed region enclosed by the bigger full ellipses. The intersection of the former region with the latter one already restricts the allowed domain. Furthermore, initial  $e^-e^+$  “ $RL$ ” polarization with  $P_1 = -P_2 = 0.8$  gives the area enclosed by the smaller solid ellipses in Fig.7a, thus further (and quite significantly) restricting the range of allowed values for the coupling constants  $\lambda_\gamma, k_\gamma$  to the shaded region. This region is magnified in Fig.7b. In Fig.8 we show the results of the same analysis, for  $\sqrt{s} = 1 \text{ TeV}$ .

In Figs.9a,b we fix, instead,  $\lambda_\gamma$  and  $k_\gamma$  at the SM values, referring again to  $\sqrt{s} = 0.5 \text{ TeV}$ . The domain allowed to  $\lambda_Z, k_Z$  by the unpolarized cross section is the interior of the dashed ellipses, while the initial “ $LR$ ” and “ $RL$ ” polarizations give the regions enclosed by the bigger and by the smaller solid ellipses, respectively. Analogous to Fig.7a, the shaded restricted domain represents the values of  $\lambda_Z, k_Z$  allowed by the combination of polarized cross sections. This domain is magnified in Fig.9b. Furthermore, in Fig.10 we repeat the same analysis, for  $\sqrt{s} = 1 \text{ TeV}$ .

We now consider the case of both initial  $e^+e^-$  and final  $W^+W^-$  polarizations, where in principle one may attempt to disentangle the anomalous coupling pairs  $(k_\gamma, k_Z)$  and  $(\lambda_\gamma, \lambda_Z)$ . Under the same input conditions leading to Figs.7–10, one finds the allowed regions displayed in Figs.11-12 for both  $W^\pm$  longitudinally polarized, for the CM energies  $\sqrt{s} = 0.5$  and  $1 \text{ TeV}$ . Specifically, the bands labeled as ‘3’ represent the regions allowed by unpolarized initial  $e^-e^+$  beams, the dashed bands ‘1’ are allowed by initial “ $RL$ ” polarization, and the dashed bands ‘2’ are the ones allowed by “ $LR$ ” polarization. As one can see, the combined measurements lead to a restricted area for the values of  $(k_\gamma, k_Z)$ , and the most restrictive case corresponds to the combination of “ $LR$ ” with “ $RL$ ”. The typical values of the bounds can be read from Fig.11-12, and are of the same order of magnitude as those indicated in the previous Section. This should be a convincing example of the essential role played by measurements of cross sections with *initial* beams polarization in improving the bounds on anomalous three-boson vertices obtainable from process (1). In particular, for  $(\lambda_\gamma, \lambda_Z)$ , one needs measurements of cross sections with both  $W^\pm$  transversely polarized, and the opportunities are qualitatively the same as the ones depicted in Fig.6, and discussed at the end of the previous Section.

The situation of the bounds would not be dramatically different from the one presented above, in the case initial longitudinal polarization was available only for the electron beam, the positron beam being unpolarized. For a right-handed electron beam ( $e_R^-$ ), the cross section would be  $\sigma^R = \sigma^{RL}/2$ , and the corresponding bounds would be higher by a factor  $\sqrt{2}$ , due to the fact that in this case the dominant uncertainty is the statistical one. For a left-handed electron beam ( $e_L^-$ ), we expect the bounds to remain almost the same as presented above, because in this case the systematic uncertainty is dominating.

Finally, we can remark that the bounds reach the order of magnitude at which anomalous vector boson couplings appear as the result of one-loop corrections [29]. In this case one should include such corrections in the cross sections appearing in Eq.(16), in order to evidence non-standard contributions to the trilinear vector boson couplings.

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## Appendix

The cross section of process (1) for arbitrary degrees of longitudinal polarization of electrons ( $P_1$ ) and positrons ( $P_2$ ) are generally expressed by Eq.(23). The corresponding polarized differential cross sections of the process  $e_b^+ e_a^- \rightarrow W_\beta^+ W_\alpha^-$  contained in Eq.(23) can be written as follows:

$$\frac{d\sigma_{\alpha\beta}^{ab}}{d\cos\theta} = C \cdot \sum_{i=0}^{i=11} F_i^{ab} \mathcal{O}_i{}_{\alpha\beta}, \quad (27)$$

where  $C = \pi\alpha_{e.m.}^2\beta_W/2s$ , the helicities of the initial  $e^-e^+$  and final  $W^-W^+$  states are labeled as  $ab = (RL, LR)$  and  $\alpha\beta = (LL, TT, TL)$ , respectively. In Eq.(27) we follow the notation used in Ref.[30]. In particular, the  $\mathcal{O}_i$  are functions of the kinematical variables which characterize to the various possibilities for the final  $W^+W^-$  polarizations ( $TT, LL, TL+LT$  or the sum of all  $W^+W^-$  polarization states for unpolarized W's). The  $F_i$  are combinations of coupling constants including the anomalous trilinear self-couplings of  $W$  bosons.

For the  $RL$  case we have:

$$\begin{aligned} F_1^{RL} &= 2(1 - g_Z g_R \cdot \chi)^2 \\ F_3^{RL} &= \Delta k_\gamma - g_Z g_R (\Delta k_\gamma + \Delta k_Z) \cdot \chi + (g_Z g_R \cdot \chi)^2 \Delta k_Z \\ F_4^{RL} &= \lambda_\gamma - g_Z g_R (\lambda_\gamma + \lambda_Z) \cdot \chi + (g_Z g_R \cdot \chi)^2 \lambda_Z \\ F_9^{RL} &= \frac{1}{2} (\Delta k_\gamma - g_Z g_R \Delta k_Z \cdot \chi)^2 \\ F_{10}^{RL} &= \frac{1}{2} (\lambda_\gamma - g_Z g_R \lambda_Z \cdot \chi)^2 \\ F_{11}^{RL} &= \frac{1}{2} [\Delta k_\gamma \lambda_\gamma - g_Z g_R (\Delta k_\gamma \lambda_Z + \Delta k_Z \lambda_\gamma) \cdot \chi + (g_Z g_R \cdot \chi)^2 \Delta k_Z \lambda_Z] \end{aligned} \quad (28)$$

The remaining  $F^{RL}$  are zero.

For the  $LR$  case we have:

$$\begin{aligned} F_0^{LR} &= \frac{1}{16s_W^4} \\ F_1^{LR} &= 2(1 - g_Z g_L \cdot \chi)^2 \\ F_2^{LR} &= -\frac{1}{2s_W^2} (1 - g_Z g_L \cdot \chi) \\ F_3^{LR} &= \Delta k_\gamma - g_Z g_L (\Delta k_\gamma + \Delta k_Z) \cdot \chi + (g_Z g_L \cdot \chi)^2 \Delta k_Z \\ F_4^{LR} &= \lambda_\gamma - g_Z g_L (\lambda_\gamma + \lambda_Z) \cdot \chi + (g_Z g_L \cdot \chi)^2 \lambda_Z \\ F_6^{LR} &= -\frac{1}{4s_W^2} (\Delta k_\gamma - g_Z g_L \Delta k_Z \cdot \chi) \\ F_7^{LR} &= -\frac{1}{4s_W^2} (\lambda_\gamma - g_Z g_L \lambda_Z \cdot \chi) \\ F_9^{LR} &= \frac{1}{2} (\Delta k_\gamma - g_Z g_L \Delta k_Z \cdot \chi)^2 \end{aligned}$$

$$\begin{aligned}
F_{10}^{LR} &= \frac{1}{2}(\lambda_\gamma - g_Z g_L \lambda_Z \cdot \chi)^2 \\
F_{11}^{LR} &= \frac{1}{2} \left[ \Delta k_\gamma \lambda_\gamma - g_Z g_L (\Delta k_\gamma \lambda_Z + \Delta k_Z \lambda_\gamma) \cdot \chi + (g_Z g_L \cdot \chi)^2 \Delta k_Z \lambda_Z \right]
\end{aligned} \quad (29)$$

The remaining  $F^{LR}$  are zero. In (28) and (29)  $s_W^2 \equiv \sin^2 \theta_W$ ;  $g_L, g_R = v \pm a$ , where  $v$  and  $a$  are the vector and axial-vector  $Ze^+e^-$  couplings ( $c_W \equiv \cos \theta_W$ ):

$$v = \frac{T_3^e - 2Q_e s_W^2}{2s_W c_W}, \quad a = \frac{T_3^e}{2s_W c_W}. \quad (30)$$

Eqs.(28)–(29) are obtained in the approximation where the imaginary part of  $Z$  boson propagator is neglected. Accounting for this effect requires the replacements  $\chi \rightarrow \text{Re}\chi$  and  $\chi^2 \rightarrow |\chi|^2$  in right-hand sides of Eqs.(28)–(29).

In Eq.(27), for the longitudinal (LL) cross sections  $\frac{d\sigma(e^+e^- \rightarrow W_L^+ W_L^-)}{d\cos\theta}$  we have (with  $|\vec{p}| = \sqrt{s}\beta_W/2$ ):

$$\begin{aligned}
\mathcal{O}_{0,LL} &= \frac{s(1 - \cos^2 \theta)}{4t^2 M_W^4} \left[ s^3(1 + \cos^2 \theta) - 4M_W^4(3s + 4M_W^2) - 4(s + 2M_W^2)|\vec{p}|s\sqrt{s}\cos\theta \right] \\
\mathcal{O}_{1,LL} &= \frac{s^3 - 12sM_W^4 - 16M_W^6}{8sM_W^4}(1 - \cos^2 \theta) \\
\mathcal{O}_{2,LL} &= \frac{1 - \cos^2 \theta}{t} \left[ \frac{|\vec{p}|s\sqrt{s}(s + 2M_W^2)}{2M_W^4} \cos\theta - \frac{s^3 - 12sM_W^4 - 16M_W^6}{4M_W^4} \right] \\
\mathcal{O}_{3,LL} &= \frac{s^2 - 2M_W^2 s - 8M_W^4}{2M_W^4}(1 - \cos^2 \theta) \\
\mathcal{O}_{4,LL} &= \mathcal{O}_{5,LL} = \mathcal{O}_{7,LL} = \mathcal{O}_{8,LL} = \mathcal{O}_{10,LL} = \mathcal{O}_{11,LL} \\
\mathcal{O}_{6,LL} &= \frac{s(1 - \cos^2 \theta)}{2tM_W^4} \left[ 8M_W^4 + 2sM_W^2 - s^2 + 2s|\vec{p}|\sqrt{s}\cos\theta \right] \\
\mathcal{O}_{9,LL} &= 2\frac{s|\vec{p}|^2}{M_W^4}(1 - \cos^2 \theta)
\end{aligned} \quad (31)$$

For the transverse (TT) cross sections  $\frac{d\sigma(e^+e^- \rightarrow W_T^+ W_T^-)}{d\cos\theta}$  we have:

$$\begin{aligned}
\mathcal{O}_{0,TT} &= \frac{4s}{t^2} \left[ s(1 + \cos^2 \theta) - 2M_W^2 - 2|\vec{p}|\sqrt{s}\cos\theta \right] (1 - \cos^2 \theta) \\
\mathcal{O}_{1,TT} &= \frac{4|\vec{p}|^2}{s}(1 - \cos^2 \theta) \\
\mathcal{O}_{2,TT} &= \frac{1 - \cos^2 \theta}{t} \left[ 4|\vec{p}|\sqrt{s}\cos\theta - 8|\vec{p}|^2 \right] \\
\mathcal{O}_{3,TT} &= \mathcal{O}_{5,TT} = \mathcal{O}_{6,TT} = \mathcal{O}_{8,TT} = \mathcal{O}_{9,TT} = \mathcal{O}_{11,TT} \\
\mathcal{O}_{4,TT} &= \frac{8|\vec{p}|^2}{M_W^2}(1 - \cos^2 \theta) \\
\mathcal{O}_{7,TT} &= \frac{s}{M_W^2} \mathcal{O}_{2,TT} \\
\mathcal{O}_{10,TT} &= \frac{4s|\vec{p}|^2}{M_W^4}(1 - \cos^2 \theta)
\end{aligned} \quad (32)$$

Finally, for the production of the one longitudinal plus one transverse vector boson ( $TL + LT$ ) we have:

$$\begin{aligned}
\mathcal{O}_{0,TL} &= \frac{2s}{t^2 M_W^2} \left[ s^2(1 + \cos^4 \theta) - 4|\vec{p}|\sqrt{s} \cos \theta (4|\vec{p}|^2 + s \cos^2 \theta) + \right. \\
&\quad \left. 4M_W^4(1 + \cos^2 \theta) + 2s(s - 6M_W^2) \cos^2 \theta - 4sM_W^2 \right] \\
\mathcal{O}_{1,TL} &= \frac{4|\vec{p}|^2}{M_W^2} (1 + \cos^2 \theta) \\
\mathcal{O}_{2,TL} &= \mathcal{O}_{6,TL} = \mathcal{O}_{7,TL} = \frac{4|\vec{p}|\sqrt{s}}{tM_W^2} \left[ (4|\vec{p}|^2 + s \cos^2 \theta) \cos \theta - 2|\vec{p}|\sqrt{s}(1 + \cos^2 \theta) \right] \\
\mathcal{O}_{3,TL} &= \mathcal{O}_{4,TL} = \mathcal{O}_{11,TL} = 2\mathcal{O}_{9,TL} = 2\mathcal{O}_{10,TL} = \frac{8|\vec{p}|^2}{M_W^2} (1 + \cos^2 \theta) \\
\mathcal{O}_{5,TL} &= \frac{32|\vec{p}|^3 \sqrt{s}}{M_W^4} \cos \theta \\
\mathcal{O}_{8,TL} &= \frac{16s|\vec{p}|^2}{tM_W^4} \left[ M_W^2 + 2|\vec{p}|\sqrt{s} \cos \theta - (s - M_W^2) \cos^2 \theta \right]
\end{aligned} \tag{33}$$



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## Figure captions

1. The  $WWV$  vertex ( $V = \gamma, Z$ ).
2. The Feynman diagrams for the  $e^+e^- \rightarrow W^+W^-$ .
3. The energy behaviour of the total SM cross sections for  $e_R^-e_L^+ \rightarrow W^-W^+$  (thin solid curve),  $e_R^-e_L^+ \rightarrow W_L^-W_L^+$  (dotted curve) and  $e_R^-e_L^+ \rightarrow W_T^-W_T^+$  (thick solid curve).
4. Qualitative features of allowed regions for  $(\Delta k_\gamma, \Delta k_Z)$  from  $e_R^-e_L^+ \rightarrow W_L^-W_L^+$  (bands '1' and '2') and  $e_L^-e_R^+ \rightarrow W_L^-W_L^+$  (bands '3' and '4').  $\delta$  is defined as:  $\frac{1}{2} \left( \frac{\delta\sigma}{\sigma} |\tilde{\mathcal{A}}| \right)_{LL}^{RL}$ .
5. Differential SM cross section for  $e^+e^- \rightarrow W^+W^-$  at  $\sqrt{s} = 500 \text{ GeV}$  for  $e^-e^+$  unpolarized (thin solid curve),  $LR$  (dotted curve) and  $RL$  (thick solid curve).
6. Allowed regions (95% C.L.) for  $(\lambda_\gamma, \lambda_Z)$  from  $e_R^-e_L^+ \rightarrow W_T^-W_T^+$  (band '1') and  $e_L^-e_R^+ \rightarrow W_L^-W_L^+$  (band '2') at  $\sqrt{s} = 500 \text{ GeV}$ .
- 7a. Allowed domains (95% C.L.) for  $(k_\gamma, \lambda_\gamma)$  for fixed  $\lambda_Z = \Delta k_Z = 0$ ;  $\sqrt{s} = 0.5 \text{ TeV}$ ;  $L_{int} = 50 \text{ fb}^{-1}$ ;  $\varepsilon_W = 0.15$ ;  $-0.9 \leq \cos \theta \leq 0.3$ . Smaller solid ellipses:  $P_1 = -P_2 = 0.8$ ; dashed ellipses:  $P_1 = P_2 = 0$ ; bigger full ellipses:  $P_1 = -P_2 = -0.8$ . Shaded allowed area: combination of polarized cross sections.
- 7b. Same as Fig.7a, magnified allowed domain.
8. Same as Fig.7b, for  $\sqrt{s} = 1 \text{ TeV}$ ;  $L_{int} = 100 \text{ fb}^{-1}$ ;  $\varepsilon_W = 0.15$ ;  $-0.9 \leq \cos \theta \leq 0.3$ .
- 9a. Similar to Fig.7a, allowed bounds (95% C.L.) for  $(k_Z, \lambda_Z)$  for fixed  $\lambda_\gamma = \Delta k_\gamma = 0$ .
- 9b Same as Fig.9a, magnified allowed domain.
10. Same as Fig.9b, for  $\sqrt{s} = 1 \text{ TeV}$ ;  $L_{int} = 100 \text{ fb}^{-1}$ ;  $\varepsilon_W = 0.15$ ;  $-0.9 \leq \cos \theta \leq 0.3$ .
11. Allowed domains (95% C.L.) for  $(k_\gamma, k_Z)$  from  $e^-e^+ \rightarrow W_L^-W_L^+$ ;  $\sqrt{s} = 0.5 \text{ TeV}$ ;  $L_{int} = 50 \text{ fb}^{-1}$ ;  $\varepsilon_W = 0.15$ ;  $-0.9 \leq \cos \theta \leq 0.3$ . Band '1':  $P_1 = -P_2 = 0.8$ ; band '2':  $P_1 = -P_2 = -0.8$ ; band '3':  $P_1 = P_2 = 0$ . Shaded allowed area: combination of polarized cross sections.
12. Same as in Fig.11, for  $\sqrt{s} = 1 \text{ TeV}$ ;  $L_{int} = 100 \text{ fb}^{-1}$ .

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